Show your work and justify all answers.

(11 pts)

(1) [**+2**] Find $\begin{bmatrix} 0 & 3 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}^{-1}$ by using Gaussian elimination.

(2) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $a, b, c, d \in \mathbb{R}$ are arbitrary.

  (a) [**+1**] Show that $AB = (ad - bc)I_2$.

  (b) [**+2**] Show that A is non-singular if and only if $ad - bc \neq 0$.

    (You may use the result of any other problem on this assignment, even if you haven't proved it)

(3) [**+1**] Suppose that $A, B \in \mathbb{R}^{n \times n}$ are both non-singular. Must $A + B$ also be non-singular?

(4) [**+2**] Let $A, B \in \mathbb{R}^{n \times n}$ where A is non-singular. Prove that $AB = O_n$ if and only if $B = O_n$. Here O_n is the $n \times n$ zero matrix.

(5) [**+3**] For a matrix $A \in \mathbb{R}^{m \times n}$, the ith row-sum of A is $\sum_{j=1}^{n} A_{ij}$ (i.e. the sum of the entries in the ith row).

  Let $A \in \mathbb{R}^{n \times n}$ be non-singular, and suppose that every row-sum of A is equal to k. What are the row-sums of A^{-1}?