Show your work and justify all answers.

(9 pts)

(1) [**+2**] Solve for $\vec{x}$ in the following linear system by finding a particular solution and the homogeneous solution(s). Write your answer in vector form, e.g. $\{\vec{u} + t\vec{v} + s\vec{w} : t, s \in \mathbb{R}\}$.

$$\begin{bmatrix} 2 & 4 & 0 \\ 2 & -2 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

(2) [**+2**] With $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 0 \end{bmatrix}$, is there any $\vec{b} \in \mathbb{R}^3$ for which $A\vec{x} = \vec{b}$ has infinitely many solutions for $\vec{x}$?

(3) [**+2**] With the same A as in problem (2), is there any $\vec{b} \in \mathbb{R}^3$ for which $A\vec{x} = \vec{b}$ does not have any solution?

(4) [**+3**] Recall that for $A \in \mathbb{R}^{m \times n}$, a matrix $R \in \mathbb{R}^{n \times m}$ is called a right-inverse of A if $AR = I_m$ and a matrix $L \in \mathbb{R}^{n \times m}$ is called a left-inverse if $LA = I_n$.

Show that $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ has infinitely many left-inverses, but does not have a right-inverse.

(5) **Bonus**[**+1**] Show that a non-square matrix $A \in \mathbb{R}^{m \times n}$ (that is, with $m \neq n$) cannot have both a left-inverse and a right-inverse.