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Problem 1. Let G be a graph and fix vertices $u, v \in V(G)$. A u, v -*disjunction* is a partition $V(G) = A \sqcup B$ with $u \in A$ and $v \in B$ such that there are no edges between A and B .

- Consider the relation $u R v$ if there is *no* u, v -disjunction in G . Show that R is an equivalence relation on $V(G)$.
- Show that there is a u, v -walk in G if and only if there is *no* u, v -disjunction in G .
- A *disjunction* in G is a partition $V(G) = A \sqcup B$ with $A, B \neq \emptyset$ such that there are no edges between A and B . Show that G is connected if and only if there is *no* disjunction in G .

Problem 2. Let G be a connected graph on at least 2 vertices. Show that if H is a proper subgraph of G , then there is some $v \in V(H)$ with $\deg_H v < \deg_G v$.

Problem 3. Show that either G or its complement $\overline{G}$ is connected.