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Problem 1. Let $f_0 = 0$, $f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ be the Fibonacci numbers. Find the generating function for the sequence a_n given by $a_n = \sum_{k=0}^n f_k f_{n-k}$.

Problem 2. Using stars&bars, we argued that $\frac{1}{(1-z)^k} = \sum_{n \geq 0} \binom{n+k-1}{k-1} z^n$. Argue the same thing by instead using the extended binomial theorem.

Problem 3. Picture an n -gon and consider dividing it into triangles by introducing $n - 3$ many non-crossing line-segments connecting pairs of vertices. Let $f(n)$ denote the number of ways to form such a triangulation. For instance, $f(3) = 1$, $f(4) = 2$ and $f(5) = 5$. For completeness, we define $f(2) = 1$. Show that $f(n)$ is a Catalan number.

Note: You don't need generating functions for this problem.

Problem 4. Let $p_d(n)$ denote the number of ways to write n as a sum of *distinct* positive integers (order doesn't matter). For examples, $p_d(4) = 2$ since the partitions are $4 = 4$ and $4 = 1 + 3$. Let $p_o(n)$ denote the number of ways to write n as the sum of *odd* positive integers. For example, $p_d(4) = 2$ since the partitions are $1 + 3$ and $1 + 1 + 1 + 1$. As usual, we define $p_d(0) = p_o(0) = 1$ for convenience.

Prove that $p_d(n) = p_o(n)$ for all $n \geq 1$.

Hint: $1 + z^n = \frac{(1+z^n)(1-z^n)}{1-z^n} = \frac{1-z^{2n}}{1-z^n}$.