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Problem 1. Use generating functions to find a closed-form formula for the recurrence $a_0 = 0$ and $a_{n+1} = 2a_n + 1$.

Problem 2. Use generating functions to find a closed-form formula for the recurrence $a_0 = 0$ and $a_{n+1} = 2a_n + n$.

Problem 3. Let a_n be the number of ways to purchase n fruits with the following restrictions:

- The number of oranges should be divisible by 5,
- The number of apples should be even,
- The number of bananas should be at most 4,
- The number of watermelons should be at most 1.

Use generating functions to compute a_n .