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Problem 1. Consider directing the edges of K_n uniformly at random. A *cyclic triangle* is a set of three vertices u, v, w such that $u \to v \to w \to u$. Let C denote the number of cyclic triangles.

- Compute $\mathbb{E}\, C$.
- Compute $\mathbf{Var}\, C$.
- Prove that if $t_n \to \infty$ as $n \to \infty$, then

$$\mathbf{Pr}\left[\left|C - \frac{1}{4}\binom{n}{3}\right| \geq t_n n^{3/2}\right] \to 0 \qquad \text{as } n \to \infty.$$

Problem 2. Use Chebyshev's inequality to prove that $\binom{2n}{n} \geq c\frac{4^n}{\sqrt{n}}$ for some constant $c > 0$. You should use only the fact that $\binom{2n}{k} \leq \binom{2n}{n}$ for any value of k.