Problem 1 (Even vs odd, revisited)**.** Use the binomial theorem to prove that, for $n \geq 1$, the number of even-sized subsets of $[n]$ is the same as the number of odd-sized subsets of $[n]$.

Problem 2 (Committee selection)**.** Find a combinatorial proof that

$$\binom{n}{r} 2^{n-r} = \sum_{k=r}^{n} \binom{n}{k}\binom{k}{r}$$

Problem 3. Let G be a (simple) graph. For vertices $x, y \in V(G)$, let $N(x, y)$ denote the set of common neighbors of x, y (that is $N(x, y) = N(x) \cap N(y)$). Prove that

$$\sum_{v \in V(G)} \binom{\deg v}{2} = \sum_{\{x,y\} \in \binom{V(G)}{2}} |N(x, y)|.$$

Problem 4 (Stirling numbers)**.** The Stirling numbers (of the second kind) $\left\{ {n \atop k} \right\}$ count the total number of ways to partition an n-element set into k non-empty pieces.

- Prove the identity
$$\left\{ {n \atop k} \right\} = \left\{ {n-1 \atop k-1} \right\} + k \left\{ {n-1 \atop k} \right\}.$$
- For a number x and a non-negative integer n, the falling-factorial is $x^{\underline{n}} = x(x-1)\cdots(x-n+1)$ with the convention that $x^{\underline{0}} = 1$ always. Prove the following binomial-theorem-esque identity:
$$x^n = \sum_{k=0}^{n} \left\{ {n \atop k} \right\} x^{\underline{k}}.$$

Problem 5. Determine, with proof, the total number of pairs (A, B) where $A \subseteq B \subseteq [n]$. For extra fun, find two different proofs: one using the binomial theorem and one using a direct bijection.