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Problem 1 (Prefix-free codes)**.** Let $[n]^{\leq k}$ denote the set of all words of length at most k with letters in $[n]$. A set $C \subseteq [n]^{\leq k}$ is said to be *prefix-free* if no element of C is a proper prefix of another element. Show that if C is prefix-free, then $|C| \leq n^k$.

Problem 2 (Bollobás's inquality)**.**

- Let A and B be disjoint sets. Consider picking an ordering of $A \sqcup B$ uniformly at random. What is the probability that every element of A appears before every element of B?
- Suppose that $A_1, \ldots, A_n$ and $B_1, \ldots, B_n$ are finite sets which satisfy $A_i \cap B_i = \varnothing$ for every i and $A_i \cap B_j \neq \varnothing$ for all $i \neq j$. Prove that

$$\sum_{i=1}^{n} \frac{1}{\binom{|A_i|+|B_i|}{|A_i|}} \leq 1.$$

- Show that part 2 implies the LYM inequality.