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Problem 1. Recall that $G_{n,p}$ is the random graph on n vertices where each edge is included independently with probability p.

- What is $\mathbb{E}|E(G_{n,p})|$?
- For $v \in V(G_{n,p})$, what is $\mathbb{E}\deg v$?
- For $u \neq v \in V(G_{n,p})$, what is $\mathbb{E}|N(u) \cap N(v)|$?

Problem 2. Consider selecting a permutation π from S_n uniformly at random and let $F(\pi)$ denote the number of fixed points of π. Determine $\mathbb{E}\,F$.

Problem 3. A *Hamilton cycle* in a directed graph is a directed cycle using every vertex. Prove that there is a way to direct the edges of the clique K_n such that there are at least $(n-1)!/2^n$ many Hamilton cycles.

Problem 4. Let $(\Omega, \mathbf{Pr})$ be a finite probability space.

- Show that if X is a random variable on Ω, then

$$\mathbb{E}\,X = \sum_{x \in X(\Omega)} x\,\mathbf{Pr}[X = x].$$

- A pair of random variables X, Y are said to be independent if the pair of events $\{X = x\}$ and $\{Y = y\}$ are independent for all $x, y \in \mathbb{R}$. Prove that if X, Y are independent random variables on Ω, then $\mathbb{E}\,XY = \mathbb{E}\,X\,\mathbb{E}\,Y$.
- Find a pair of random variables X, Y for which $\mathbb{E}\,XY \neq \mathbb{E}\,X\,\mathbb{E}\,Y$.
- Find a pair of random variables X, Y which are *not* independent, yet $\mathbb{E}\,XY = \mathbb{E}\,X\,\mathbb{E}\,Y$.