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Problem 1. Let G be a bipartite graph. Prove that G is regular if and only if we can color the edges of G so that each color class is a perfect matching in G .

Problem 2. Fix $k \leq \frac{n-1}{2}$. Show that there is an injection $f: \binom{[n]}{k} \rightarrow \binom{[n]}{k+1}$ with $A \subseteq f(A)$ for all $A \in \binom{[n]}{k}$.

Problem 3. Sportsball is a game played between two teams that cannot end in a tie. We have a sportsball league with $2n$ teams which plays over a season of $2n - 1$ days. Multiple games can take place per day, but each team plays at most one game per day. Furthermore, over the course of the season, each team plays every other team exactly once.

At the end of the season, we have $2n - 1$ trophies to hand out, one for each day. The trophy for day i must go to one of the winning teams on day i . Furthermore, each team should get at most one trophy. Prove that there is a way to hand out all $2n - 1$ trophies.

Hint: For a set of days S , consider fixing a team t that did not win on a day in S . Show that there is an injection from the rest of the other teams that did not win on a day in S to days *not* in S .