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Problem 1. For any graph G , show that $|E(G)| \geq \binom{\chi(G)}{2}$.

Problem 2. A graph G is called k -critical if $\chi(G) \geq k$ yet $\chi(H) \leq k - 1$ for every proper subgraph $H \subseteq G$.

- Show that if G is k -critical, then actually $\chi(G) = k$.
- Show that the only 2-critical graph is K_2 .
- Show that the only 3-critical graphs are the odd cycles.
- Show that if $\chi(G) \geq k$, then G has a k -critical subgraph.
- Show that if G is k -critical, then $\delta(G) \geq k - 1$.