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Problem 1. G is a non-bipartite graph such that every proper subgraph of G is bipartite. Show that G must be an odd cycle.

Problem 2. Show that if $\delta(G) \geq 2$, then G contains a cycle.

Hint: Start with a maximum-length path in G .

Problem 3. The *independence number* $\alpha(G)$ is the size of the largest independent set in G . Prove that $\alpha(C_n) = \lfloor \frac{n}{2} \rfloor$.

Problem 4. Show that $\alpha(G) \geq \frac{|V(G)|}{\Delta(G)+1}$ for any graph G .