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Solutions must be typeset in $\text{\LaTeX}$. Collaboration is allowed, but must be fully acknowledged. Any collaboration should involve only discussion; all writing **must** be done individually. Use of generative AI is **forbidden**. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

Problem 1. 61 points are placed in a disk of radius 4. Prove that some pair of points are at most $\sqrt{2}$ units apart.

Problem 2. $n + 1$ integers are chosen from the set $[2n]$. Prove that one of these integers divides another.

Hint: Every positive integer can be written as $2^a b$ where $a \geq 0$ and b is odd.

Problem 3. Let G be a (simple) graph on at least 2 vertices. Show that G has some pair of vertices of the same degree.

Problem 4. Fix $n \geq 3$ and suppose that $S \subseteq [n]$ with $|S| \geq 2 \log_2 n$. Show that there are two distinct subsets $A, B \subseteq S$ with $\sum A = \sum B$.