This document is from https://mathematicaster.org/teaching/discrete2026/hw/hw11.pdf

Solutions must be typeset in LaTeX. Collaboration is allowed, but must be fully acknowledged. Any collaboration should involve only discussion; all writing **must** be done individually. Use of generative AI is **forbidden**. You are free to use the result from any problem on this (or previous) assignment as a part of your solution to a different problem even if you have not solved the former problem.

Problem 1. Consider the random graph $G_{n,1/2}$. Show that if $t_n \to \infty$ as $n \to \infty$, then

$$\mathbf{Pr}\left[\forall v \in V,\ \left|\deg v - \frac{n-1}{2}\right| \le t_n\sqrt{n\log n}\right] \to 1 \qquad \text{as } n \to \infty.$$

Hint: Chernoff + union bound.

Problem 2. Suppose that $\epsilon_1, \dots, \epsilon_n$ are sampled independently and uniformly from $\{\pm 1\}$ and set $S_n = \epsilon_1 + \cdots + \epsilon_n$. Prove that there are constants $C, c > 0$ such that

$$c\sqrt{n} \le \mathbb{E}|S_n| \le C\sqrt{n}.$$

Hint for the lower-bound: Markov states that $\mathbb{E}|X| \ge t \cdot \mathbf{Pr}[|X| \ge t]$; consider optimizing over the choice of t here. Also, remember Stirling's approximation implies that $\binom{n}{n/2} \le C\frac{2^n}{\sqrt{n}}$ for some constant C.

Problem 3. Let $G_1, \dots, G_k$ be graphs on the same vertex-set V, where G_i has m_i many edges. Prove that there is a common bipartition $V = A \sqcup B$ such that each G_i has at least $\frac{m_i}{2} - \sqrt{km_i}$ many edges crossing between A and B.

Hint: Chebyshev + union bound.