These notes are from http://math.cmu.edu/~cocox/teaching/discrete20/rec12.pdf

Let G be a graph and let $\chi\colon E \to \{0,1\}$ be a 2-coloring of the edges of G. In this coloring, we can define $\deg_i^\chi(v) = \big|\{u \in V : \chi(uv) = i\}\big|$ for $i \in \{0,1\}$; that is, $\deg_i^\chi(v)$ is the number of edges of color i incident to v in the coloring χ. For $\chi\colon E \to \{0,1\}$, we define the *discrepancy of* χ to be

$$\mathrm{disc}(G,\chi) \stackrel{\text{def}}{=} \max_{v\in V}\big|\deg_0^\chi(v) - \deg_1^\chi(v)\big|.$$

Of course, for any graph G and 2-coloring χ, we have $\mathrm{disc}(G,\chi) \le \Delta(G)$, which is tight when χ gives every edge the same color. So the interesting question is to try to minimize the discrepancy; hence, define

$$\mathrm{disc}(G) \stackrel{\text{def}}{=} \min_{\chi\colon E\to\{0,1\}} \mathrm{disc}(G,\chi).$$

Observe that if G has a vertex of odd degree, then $\mathrm{disc}(G) \ge 1$. Moreover, observe that if G is a cycle with an odd number of vertices, then no matter how the edges are 2-colored, there must be some vertex incident to edges of only a single color, so $\mathrm{disc}(G) = 2$. It turns out that an odd cycle is a worst-case scenario (see the exercise below for a classification of all worst-case scenarios).

Claim 1. *For any graph G,* $\mathrm{disc}(G) \le 2$.

Proof. We will complete the proof in a couple steps.

Step 1: G is connected and $\deg(v)$ is even for every $v \in V$. Here we prove something slightly stronger: For any fixed $v \in V$, there is a coloring $\chi\colon E \to \{0,1\}$ such that $|\deg_0^\chi(v) - \deg_1^\chi(v)| \le 2$ and $\deg_0^\chi(u) = \deg_1^\chi(u)$ for all $u \ne v$.

Since G is connected and every vertex has even degree, we know that G has an Eulerian walk. We may suppose that this walk starts at v: label the edges $e_1, e_2, \dots, e_m$ in the order they're traversed and define $\chi(e_i) = i \mod 2$. Consider any vertex $u \ne v$. Observe that if $u \in e_i$, then either $u \in e_{i-1}$ or $u \in e_{i+1}$ (but not both). Hence, we can pair up edges of opposite color incident to u, implying that $\deg_0^\chi(u) = \deg_1^\chi(u)$. The same logic almost works for v, except for the fact that e_1, e_m are paired up and these edges may have the same color. Hence, we have $\deg_0^\chi(v) = \deg_1^\chi(v)$ if m is even and $\deg_1^\chi(v) = \deg_0^\chi(v) + 2$ if m is odd.[1]

Step 2: G is connected and some vertex has odd degree. Let $U \subseteq V$ denote the set of vertices of G with odd degree. We build a new graph G' from G by adding a new vertex which is connected to each vertex of U. Formally, $V(G') = V \cup \{v'\}$ and $E(G') = E \cup \{\{v', u\} : u \in U\}$. The handshaking lemma tells us that $|U|$ is even, so since $|U| \ge 1$ by assumption, we know that G' is connected and every vertex of G' has even degree.

By step 1, we may find a coloring $\chi'\colon E(G') \to \{0,1\}$ such that $|\deg_0^{\chi'}(v') - \deg_1^{\chi'}(v')| \le 2$ and $\deg_0^{\chi'}(v) = \deg_1^{\chi'}(v)$ for all $v \in V(G') \setminus \{v'\} = V$. Thus, by deleting v' and its incident edges, we are left with a coloring $\chi\colon E \to \{0,1\}$ for which $|\deg_0^\chi(v) - \deg_1^\chi(v)| \le 1$ for all $v \in V$.[2]

[1] Observe that we've proved something more than claimed here: If G is a connected graph with an even number of edges and every vertex has even degree, then $\mathrm{disc}(G) = 0$.

[2] Observe that we've proved something more than claimed here: If G is a connected graph with at least one vertex of odd degree, then $\mathrm{disc}(G) = 1$.

Step 3: G is arbitrary. We can decompose G into $G = G_1 \cup G_2 \cup \cdots \cup G_k$ where the G_i's are vertex disjoint and each G_i is connected. Observe that since the G_i's are vertex disjoint, $\operatorname{disc}(G) = \max_{i \in [k]} \operatorname{disc}(G_i)$. Putting together steps 1 and 2, since each G_i is connected, $\operatorname{disc}(G_i) \leq 2$, and so $\operatorname{disc}(G) \leq 2$ as well. □

Exercise. Let G be a connected graph. Show that $\operatorname{disc}(G) = 2$ if and only if G has an odd number of edges and every vertex has even degree.

Exercise. Let G be a connected graph and suppose that $\chi \colon E \to \{0, 1\}$ is any 2-coloring for which $\operatorname{disc}(G, \chi) = 0$. Show that G has an Eulerian walk $e_1, \ldots, e_m$ such that $\chi(e_i) = i \mod 2$.