This quiz is from http://math.cmu.edu/~cocox/teaching/discrete20/quiz14.pdf

Problem 1. Let $G = (V, E)$ be a connected graph and let $f : V \to X$ be any function (where X is any arbitrary set). Prove that either f is a constant function (i.e. $|f(V)| = 1$) or that there is an edge $\{u, v\} \in E$ where $f(u) \neq f(v)$. Show also that the connectivity assumption is crucial.
(Note: This fact pops up time and time again, so it's worth keeping in mind!)

Problem 2. Let $G = (V, E)$ be a connected graph and let S be any subset of E which does not contain a cycle. Prove that G has a spanning tree which uses every edge of S. In other words, any acyclic set of edges can be extended to a spanning tree.

Problem 3. Let T, F be trees on the same vertex set. For any edge $e \in E(T) \setminus E(F)$, we know that $F + e$ contains a unique cycle: call this cycle C_e. Prove that

$$E(F) \setminus E(T) \subseteq \bigcup_{e \in E(T) \setminus E(F)} E(C_e).$$

Problem 4. Let $G = (V, E)$ be a weighted graph with weight function $w : E \to \mathbb{R}$. Suppose that G is connected and every edge has a distinct weight under w (i.e. $w(e) \neq w(s)$ for all $e \neq s \in E$). Prove that G has a *unique* minimum spanning tree.