This quiz is from http://math.cmu.edu/~cocox/teaching/discrete20/quiz13.pdf

Just so that there's no confusion, every graph in this quiz is assumed to be simple. That is, graphs cannot contain loops nor multiple edges between two vertices.

Problem 1. We say that a graph $G = (V, E)$ is 2-*edge-connected* if $G - e$ is connected for every $e \in E$. Show that if G is a connected graph wherein each vertex has even degree, then G is 2-edge-connected.

Problem 2. Let $T = (V, E)$ be a tree on at least 2 vertices. Let $\ell(T)$ denote the number of leaves of T. Prove that

$$\ell(T) = 2 + \sum_{\substack{v \in V:\\ \deg(v) \geq 2}} \big(\deg(v) - 2\big).$$

Problem 3. Does there exist a planar graph G which is both triangle-free and has $\delta(G) \geq 4$?

Problem 4. Let G be a planar graph on n vertices. Prove that G has at most $3n$ edges.

Problem 5 (Bonus)**.** For a positive integer k and a graph $G = (V, E)$, a coloring $\chi\colon V \to [k]$ is called a *proper* k*-coloring* if $\chi(u) \neq \chi(v)$ whenever $\{u, v\} \in E$ (i.e. adjacent vertices get different colors).

Prove that every triangle-free planar graph has a proper 4-coloring. (This is a special case of the famous four color theorem)