This quiz is from http://math.cmu.edu/~cocox/teaching/discrete20/quiz11.pdf

Problem 1. Let X be a random variable on a finite or countable probability space $(\Omega, \mathbf{Pr})$ such that $\mathbb{E}\,X$ is finite. Show that there exist $\omega, \omega' \in \Omega$ for which $X(\omega) \leq \mathbb{E}\,X \leq X(\omega')$.

Problem 2. State and prove Markov's inequality.

Problem 3. State and prove Chebyshev's inequality.

Problem 4. Suppose a coin is biased so that $\mathbf{Pr}[H] = p$ and $\mathbf{Pr}[T] = 1 - p$ for some fixed $p \in (0,1)$. Consider repeatedly flipping this coin (flips are independent) until we see a T. Let X be the random variable which counts the number of heads in the experiment (e.g. $X(HHHT) = 3$ and $X(T) = 0$). Compute $\mathbb{E}\,X$.

Problem 5 (Bonus)**.** Recall property S_k from HW7(1). Let $n(k)$ denote the least integer n for which there is a tournament with n teams which has property S_k. Prove that

$$2^{k+1} - 1 \leq n(k) \leq k^2 2^{k+2}.$$