

(Rewrite) [+3] Let φ be a root of the equation $X^2 - X = 1$, that is to say $\varphi^2 = 1 + \varphi$. Prove that $\varphi^n = \varphi \cdot f_n + f_{n-1}$ for all $n \in \mathbb{N}$. (Note: it is not necessary to calculate what φ actually is.)