Let $f : \mathbb{R} \to \mathbb{R}$. We say that f is *increasing* if $f(x) \geq f(y)$ whenever $x \geq y$. Similarly f is *decreasing* if $f(x) \leq f(y)$ whenever $x \geq y$.

(25 pts)

(1) Let $f, g : \mathbb{R} \to \mathbb{R}$. Prove or disprove the following statements.
- (a) [+3] If f, g are increasing, then $f \circ g$ is increasing.
- (b) [+3] If f is increasing and g is decreasing, then $f \circ g$ is decreasing.
- (c) [+3] If f is increasing and g is decreasing, then $f - g$ is increasing.
- (d) [+3] If f, g are increasing, then fg is increasing.

(2) For sets A, B, define the *symmetric difference* to be $A \triangle B := (A \setminus B) \cup (B \setminus A)$.
- (a) [+5] Prove that $A \triangle B = (A \cup B) \setminus (A \cap B)$.

(3) [+8] Which of the following is true? Prove your claim.
- (a) $\mathcal{P}(\mathbb{N}) \subsetneq \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$
- (b) $\mathcal{P}(\mathbb{N}) \supsetneq \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$
- (c) $\mathcal{P}(\mathbb{N}) = \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$.