In this assignment, we will explore chessboard tilings. A domino covers exactly two squares of a chessboard that are either horizontally or vertically adjacent. A proper tiling of a board is one where each square is covered exactly one domino. For example the following are proper tilings of the $2 \times 4$ board with dominoes

![Proper Tiling Example](image)

whereas these are not

![Improper Tiling Example](image)

Justify all answers! If a tiling is possible, exhibit one or give a process which will create one. If a tiling is impossible, you must prove this.

(27 pts)

1. [+4] For what values of $m$ and $n$ does an $m \times n$ board have a domino tiling?

2. [+8] Consider the $8 \times 8$ board with two opposite corners removed:

![Modified Board](image)

Does this board have a domino tiling?

3. Consider the following object called a tetromino:

![Tetromino](image)

(a) [+5] Does the $8 \times 8$ board have a tetromino tiling? (Note: the tetromino can be rotated, and we are talking about the standard $8 \times 8$ board and not the modified one in the previous question.)

(b) [+10] How about the $10 \times 10$ board?

4. **Bonus.** [+4] A domino tiling is said to be *stable* if every horizontal and vertical line drawn through the board must cross some domino. For example, the first tiling is stable, while the second is not.

![Stable and Unstable Tilings](image)

Does the $6 \times 6$ board have a stable domino tiling?