Justify all answers!

## (6 pts)

(1) [+4] Let $f : A \to B$ and let $S, T \subseteq B$. Prove that

$$\text{PreIm}_f(S \cup T) = \text{PreIm}_f(S) \cup \text{PreIm}_f(T).$$

*Proof.* ($\subseteq$) Let $x \in \text{PreIm}_f(S \cup T)$ be arbitrary. Therefore, there is some $y \in S \cup T$ with $f(x) = y$. We break into cases: if $y \in S$, then $x \in \text{PreIm}_f(S)$ by definition. On the other hand, if $y \in T$, then $x \in \text{PreIm}_f(T)$. In either case, $x \in \text{PreIm}_f(S) \cup \text{PreIm}_f(T)$, so as $x$ was arbitrary, $\text{PreIm}_f(S \cup T) \subseteq \text{PreIm}_f(S) \cup \text{PreIm}_f(T)$.

($\supseteq$) Let $x \in \text{PreIm}_f(S) \cup \text{PreIm}_f(T)$ be arbitrary. We again break into cases. If $x \in \text{PreIm}_f(S)$, then there is some $y \in S$ for which $f(x) = y$. As $y \in S \subseteq S \cup T$, we attain $x \in \text{PreIm}_f(S \cup T)$. Similarly, if $x \in \text{PreIm}_f(T)$, then there is some $z \in T$ with $f(x) = z$. As $z \in T \subseteq S \cup T$, we again attain $x \in \text{PreIm}_f(S \cup T)$. Thus, as $x$ was arbitrary, we have $\text{PreIm}_f(S) \cup \text{PreIm}_f(T) \subseteq \text{PreIm}_f(S \cup T)$. □

(2) [+2] Let $f : A \to B$ and let $S, T \subseteq A$. Prove or disprove that

$$\text{Im}_f(S \setminus T) \subseteq \text{Im}_f(S) \setminus \text{Im}_f(T).$$

*Proof.* The statement is false, as shown by the following counterexample. Let $A = \{a, b\}$, $B = \{1\}$ and $g : A \to B$ be defined by $g = \{(a, 1), (b, 1)\}$. Letting $S = \{a\}$ and $T = \{b\}$, we find

$$\text{Im}_f(\{a\} \setminus \{b\}) = \text{Im}_f(\{a\}) = \{1\};$$

however,

$$\text{Im}_f(\{a\}) \setminus \text{Im}_f(\{b\}) = \{1\} \setminus \{1\} = \varnothing.$$ □

(Rewrite) [+3] Let $f : A \to B$ and let $S, T \subseteq B$. Prove that

$$\text{PreIm}_f(S \setminus T) \subseteq \text{PreIm}_f(S) \setminus \text{PreIm}_f(T).$$

*Proof.* Let $x \in \text{PreIm}_f(S \setminus T)$ be arbitrary. Therefore, $f(x) \in S \setminus T$, so $f(x) \in S$ and $f(x) \notin T$. Thus, $x \in \text{PreIm}_f(S)$ and $x \notin \text{PreIm}_f(T)$, so $x \in \text{PreIm}_f(S) \setminus \text{PreIm}_f(T)$. □