Justify all answers!

(8 pts)

(1) [+4] Let S and T be sets whose elements are sets, themselves. For example, maybe S and T are powersets. Prove that if $S \subseteq T$, then

$$\bigcup_{X\in S} X \subseteq \bigcup_{Y\in T} Y.$$

Proof. Let $x \in \bigcup_{X\in S} X$ be arbitrary. By the definition of union, there is some $X \in S$ for which $x \in X$. Now, as $S \subseteq T$ and $X \in S$, we know $X \in T$. Thus, there is some $Y \in T$, namely $Y = X$, for which $x \in Y$, so $x \in \bigcup_{Y\in T} Y$. □

(2) [+4] For sets A, B, C, D, prove that $(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$.

Proof. Let $x \in (A \times B) \cap (C \times D)$ be arbitrary, then $x \in A \times B$ and $x \in C \times D$. This means that $x = (y, z)$ where $y \in A, z \in B$ and $y \in C, z \in D$. Therefore $y \in A \cap C$ and $z \in B \cap D$. In other words, $x = (y, z) \in (A \cap C) \times (B \cap D)$. □

(Rewrite) [+3] For sets A, B, C, D, prove that $(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D)$.

Proof. Let $(x, y) \in (A \cap C) \times (B \cap D)$ be arbitrary. Thus, $x \in A \cap C$ and $y \in B \cap D$. In particular, $x \in A$ and $y \in B$, so $(x, y) \in A \times B$. Similarly, $x \in C$ and $y \in D$, so $(x, y) \in C \times D$. Therefore, $(x, y) \in (A \times B) \cap (C \times D)$. □