If you are TEXing, the symbol " $\boldsymbol{.}$ " can be made by adding the command "`\newcommand{\st}{\,\hbox{\Huge .}\,}`" to the preamble and then called by "`\st`" when in math mode.

(10 pts)

(1) For each of the following propositions, write the negation without using $\neg$

(a) [+2] $\forall x \in \mathbb{Z} \,.\, \exists y \in \mathbb{Z} \,.\, x = 2y$

Proof. $\exists x \in \mathbb{Z} \,.\, \forall y \in \mathbb{Z} \,.\, x \neq 2y$ □

(b) [+2] $\exists y \in \mathbb{R} \,.\, \forall x \in \mathbb{R} \,.\, y > x$

Proof. $\forall y \in \mathbb{R} \,.\, \exists x \in \mathbb{R} \,.\, y \leq x$ □

(c) [+2] $\forall \epsilon > 0 \,.\, \exists \delta > 0 \,.\, (|x - x_0| < \delta) \Rightarrow (|y - y_0| < \epsilon)$

Proof. $\exists \epsilon > 0 \,.\, \forall \delta > 0 \,.\, (|x - x_0| < \delta) \wedge (|y - y_0| \geq \epsilon)$ □

(2) Write the contrapositive of the following statements.

(a) [+2] For integers x, y, z, if x divides y and x divides z, then x divides $y + z$.

Proof. For integers x, y, z, if x does not divide $y + z$, then x does not divide y or x does not divide z. □

(b) [+2] For real numbers x, y, whenever $x \geq y$, it must be the case that $x^2 \geq y^2$.

Proof. For real numbers x, y, if $x^2 < y^2$, then $x < y$. □

(Rewrite) [+2] Write the contrapositive of the following statement: For any $a, b \in \mathbb{R}$, if $a > b$, then there is $\epsilon > 0$ for which $a > b + \epsilon$.

Proof. For any $a, b \in \mathbb{R}$, if $a \leq b + \epsilon$ for every $\epsilon > 0$, then $a \leq b$. □