Justify all answers!

(6 pts)

(1) Find an injection $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$.

(a) [+4] Prove that your function is injective.

Proof. There are many functions that work, but let's take $f(x) = \{x\}$ for all $x \in \mathbb{N}$. Certainly $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$ as $\{x\} \subseteq \mathbb{N}$ for all $x \in \mathbb{N}$.

Now, suppose that $f(x) = f(y)$ for some $x, y \in \mathbb{N}$, then $\{x\} = \{y\}$, so we must have $x = y$. Thus, f is an injection. □

(b) [+2] Prove that your function is *not* surjective.[1]

Proof. Note that for any $x \in \mathbb{N}$, $f(x) = \{x\} \neq \varnothing$. Thus, f is not surjective as $\varnothing \in \mathcal{P}(\mathbb{N})$ is not mapped to by f. □

(Rewrite) [+2] Consider $f : \mathbb{N} \times \mathbb{N} \to \mathcal{P}(\mathbb{N})$ defined by $f(a, b) = \{a + 2, 2b\}$. Is f an injection? Prove your claim.

Proof. No, f is not an injection. As an example, $f(2, 3) = \{4, 6\}$ and $f(4, 2) = \{6, 4\}$, so $f(2, 3) = f(4, 6)$ while $(2, 3) \neq (4, 6)$. □

[1] Yes! Somehow I know that no matter what function you picked, it won't be surjective!