

In this homework, we will prove a fact that we've all known since grade school, namely, a trick to determine whether a number is divisible by 3. Let $a = "a_n a_{n-1} \cdots a_1 a_0"$ be a number where a_i is the i th digit of a when written in decimal form; in particular, $a = \sum_{i=0}^n a_i \cdot 10^i$. For example if $a = 1045$, then $a_0 = 5$, $a_1 = 4$, $a_2 = 0$ and $a_3 = 1$.

Justify all answers!

(6 pts)

- (1) [+4] Let $a = "a_n a_{n-1} \cdots a_1 a_0"$ be an integer where a_i is the i th digit of the decimal form of a . Prove that $3 \mid a$ if and only if $3 \mid \sum_{i=0}^n a_i$. (Hint: modular arithmetic may be helpful)
- (2) [+2] Let $n \in \mathbb{N}$ have $n \geq 2$. Prove that $(n-1)^{-1} \equiv n-1 \pmod{n}$.