Justify all answers!

(4 pts)

(1) [+4] Let $x, y, z \in \mathbb{N}$ where $\gcd(x, y) = 1$. Without using the fundamental theorem of arithmetic, prove that if $x \mid z$ and $y \mid z$, then $xy \mid z$.

Proof. As $x \mid z$, there is some $c \in \mathbb{Z}$ with $z = cx$. Thus, as $y \mid z$, we have $y \mid cx$, so because $\gcd(y, x) = 1$, it must be the case that $y \mid c$, as shown in class. As such, there is some $d \in \mathbb{Z}$ with $c = yd$. As such $z = cx = dxy$, which means that $xy \mid z$. □

(Rewrite) [+2] Let $a, b \in \mathbb{N}$. Prove that if $\gcd(ab, a + b) = 1$, then $\gcd(a, b) = 1$.

Proof. As $\gcd(ab, a + b) = 1$, by Bezout's identity, there is $x, y \in \mathbb{Z}$ with $abx + (a + b)y = 1$. By rearranging, we have $a(bx + y) + by = 1$, so as $bx + y, y \in \mathbb{Z}$, we have $\gcd(a, b) = 1$. □