Justify all answers with formal arguments!

(12 pts)

(1) (a) [+3] Fill in the following truth table

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$(\neg p) \wedge (\neg q)$
T	T					
T	F					
F	T					
F	F					

(b) [+1] What logical equivalence does this truth table prove?

(2) For two n-sided dice A, B, we say that $A \asymp B$ if the probability that A rolls higher than B is equal to the probability that B rolls higher than A; in other words, $A \asymp B$ if, on average, A and B "tie."[1]

(a) [+2] Show that $A \asymp A$ for any die A.

(b) [+6] Is "$\asymp$" transitive? In other words, if we have n-sided dice A, B, C with $A \asymp B$ and $B \asymp C$, must it be the case that $A \asymp C$?

(3) **BONUS.** [+3] The standard n-sided die, which we denote by S_n, is an n-sided die with sides labeled $1, 2, 3, \ldots, n$, just as you would expect. Now, fix integers $d_1, \ldots, d_n$ (not necessarily distinct) between 1 and n, inclusive, such that $d_1 + \cdots + d_n = \frac{n(n+1)}{2}$. If D is the n-sided die with sides labeled $d_1, \ldots, d_n$, must it be true that $D \asymp S_n$? Feel free to use the formula $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, which we will prove later.

[1]Remember, these are independent, fair dice, so the probability that A rolls higher than B is simply

$$\frac{\text{the number of ways for } A \text{ to roll higher than } B}{n^2}$$