

Problem 1. A farmer's field is a disk with a 1 mile radius. Within the field, the farmer wishes to build a rectangular pen which encloses as much area as possible. How should the farmer build the pen?

Problem 2. A steel pipe is being carried down a hallway which is A feet wide. At the end of the hall, there is a right-angle turn into a narrower hallway, which is B feet wide. What is the length of the longest pipe that can be carried horizontally around the corner (possibly while terribly scratching the walls)?

(The answer can be expressed in the form $(A^\alpha + B^\alpha)^{1/\alpha}$ for some $\alpha > 0$)

Problem 3. A clock has a 4 cm long minute-hand and a 3 cm long hour-hand. How quickly is the distance between the hour-hand and the minute-hand (measured between their tips) changing at 2:00?

(Note: It's a standard analogue clock, so both hands are continuously moving)

Problem 4. Consider all circles of radius 5 which are tangent to both $y = \frac{3}{4}x - 6$ and $y = -\frac{3}{4}x + 9$. Find the center of the left-most of these circles.

Problem 5. $f(x)$ is a differentiable function defined for $0 < x < \infty$ which has $f(1) = 1$. Furthermore, for any constant C , if the curves $y = f(x)$ and $y = x^2 + C$ intersect, then they are perpendicular at this intersection point (that is, the tangent lines are perpendicular). Determine the function $f(x)$.